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TOPSIS in Business Analytics

William P. Fox
Naval Postgraduate School, USA

INTRODUCTION

Multi-attribute decision making (MADM) and multi-criteria decision making (MCDM) problems with *m* alternatives that are evaluated by *n* attributes may be viewed as a geometric system with *m* points in *n* -dimensional space. Hwang and Yoon (1981) developed the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) based on the concept that the chosen alternative should have the shortest distance from the positive-ideal solution (PIS) and the longest distance from the negative-ideal solution (NIS). This principle has been also suggested by Zeleny (1982) and Hall (1989), and it has been enriched by Yoon (1987) and Hwang, Lai, and Liu (1993). Further discussion was made by many (Chu, 2002; Olson, 2004; Peng, 2000). The PIS has the best measures over all attributes, while the NIS has the worst measures over all attributes (Wu, 2006). An ideal solution is defined as a collection of ideal levels (or ratings) in all attributes considered. It is assumed that the true ideal solution is usually unattainable or infeasible so to be as close as possible to such an ideal solution is the rationale of human choice. TOPSIS is one of the most popular MCDM methods (Ozturk, 2011).

In this chapter we describe the methodology for TOPSIS, provide a few examples in decision making each to illustrate TOPSIS, briefly mention the role of technology that might be used in obtaining solution, and the interpretation of the TOPSIS solution. We present some the strengths and weaknesses to the process.

BACKGROUND

TOPSIS was the result of work done by Yoon and Hwang (1980). TOPSIS has been used in a wide spectrum of comparisons of alternatives including: item selection from among alternatives, ranking leaders or entities, remote sensing in regions, data mining, and supply chain operations. TOPSIS is chosen over other methods because it orders the feasible alternatives according to their closeness to an ideal solution (Malezewski, 1996).

Napier (1992) provided some analysis of the use of TOPSIS for the department of defense in industrial base planning and item selection. For years the military used TOPSIS to rank order the systems' request from all the branches within the service for the annual budget review process (Fox, 2012) as well as being taught again in as part of decision analysis. Current work is being done to show the ability of TOPSIS to rank order nodes of a dark or social network across all the metrics of social network analysis (Fox, 2012; Fox & Everton, 2013).

In manufacturing analysis, Wang et al. (2008) proposed two methods to improve TOPSIS for multi-response optimization using Taguchi's loss function. Ozturk and Batuk (2011) used TOPSIS for spatial decisions and then linked to geographical information systems (GIS) operations for flood vulnerability. Olson and Wu (2005, 2006) have shown how TOPSIS may be used for data mining and analysis in credit card score data. Olson (2006) presented a comparison of weights (centroid weights, equal weights, and weights by linear regression) in TOPSIS models using baseball data where their conclusion is that accurate weights in TOPSIS are crucial to success.

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In a business setting it has been applied to a large number of application cases in advanced manufacturing processes (Agrawal, Kohli, & Gupta, 1991; Parkan & Wu, 1999), purchasing and outsourcing (Kahraman, Engin, Kabak, & Kaya, 2009; Shyura & Shih, 2006), and financial performance measurement (Feng & Wang, 2001).

In social networks, TOPSIS has been used to rank order the nodes across all metrics in order to identify the most influential node (Fox, et al. 2013)

MAIN FOCUS OF THE CHAPTER: TOPSIS

TOPSIS Methodology

The TOPSIS process is carried out as follows:

Step 1: Create an evaluation matrix consisting of *m* alternatives and *n* criteria, with the intersection of each alternative and criteria given as x_{ij}, giving us a matrix $(\boldsymbol{X}_{ij})_{mxn}$.

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \\ A_m \end{array} \begin{bmatrix} x_1 & x_2 & x_3 & x_n \\ x_{11} & x_{12} & x_{13} & x_{1n} \\ x_{21} & x_{22} & x_{23} & x_{2n} \\ x_{31} & x_{32} & x_{33} & x_{3n} \\ & & & \\ x_{m1} & x_{m2} & x_{m3} & x_{mn} \end{bmatrix}$$

Step 2: The matrix shown as ***D*** above then normalized to form the matrix $\boldsymbol{R}=(\boldsymbol{R}_{ij})_{mxn}$, using the normalization method

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum x_{ij}^2}}$$

for *i=1,2…,m; j= 1,2,…n*

Step 3: Calculate the weighted normalized decision matrix. First we need the weights. Weights can come from either the decision maker or by computation.

Step 3a: Use either the decision maker's weights for the attributes $x_1, x_2, ..x_n$ or compute the weights through the use Saaty's (1980) AHP's decision maker weights method to obtain the weights as the eigenvector to the attributes versus attribute pair-wise comparison matrix.

$$\sum_{j=1}^{n} w_j = 1$$

The sum of the weights over all attributes must equal 1 regardless of the method used.

Step 3b: Multiply the weights to each of the column entries in the matrix from *Step 2* to obtain the matrix, ***T***.

$$T = (t_{ij})_{m\,x\,n} = (w_j r_{ij})_{m\,x\,n}, i = 1,2,\ldots,m$$

Step 4: Determine the worst alternative (A_w) and the best alternative (A_b): Examine each attribute's column and select the largest and smallest values appropriately. If the values imply larger is better (profit) then the best alternatives are the largest values and if the values imply smaller is better (such as cost) then the best alternative is the smallest value.

$$A_w = \\ \left\{ \max(t_{ij} | i = 1,2,\ldots,m \mid j \in J_-, \min(t_{ij} | i = 1,2,\ldots,m) \mid j \in J_+ \right\} \equiv \\ \left\{ t_{wj} | j = 1,2,\ldots,n \right\},$$

$$A_{wb} = \\ \left\{ \min(t_{ij} | i = 1,2,\ldots,m \mid j \in J_-, \max(t_{ij} | i = 1,2,\ldots,m) \mid j \in J_+ \right\} \equiv \\ \left\{ t_{bj} | j = 1,2,\ldots,n \right\},$$

where,

$J_{+} = \{j = 1,2,\ldots n|j)$ associated with the criteria having a positive impact, and
$J_{-} = \{j = 1,2,\ldots n|j)$ associated with the criteria having a negative impact.

We suggest that if possible make all entry values in terms of positive impacts.

Step 5: Calculate the L2-distance between the target alternative *i* and the worst condition A_w

$$d_{iw} = \sqrt{\sum_{j=1}^{n}(t_{ij} - t_{wj})^2}, i=1,2,\ldots m$$

and the distance between the alternative *i* and the best condition A_b

$$d_{ib} = \sqrt{\sum_{j=1}^{n}(t_{ij} - t_{bj})^2}, i=1,2,\ldots m$$

where d_{iw} and d_{ib} are L2-norm distances from the target alternative *i* to the worst and best conditions, respectively.

Step 6: Calculate the similarity to the worst condition:

$$s_{iw} = \frac{d_{iw}}{\left(d_{iw} + d_{ib}\right)}, 0 \le s_{iw} \le 1, i = 1,2,\ldots,m$$

.$S_{iw}=1$ if and only if the alternative solution has the worst condition; and
$S_{iw}=0$ if and only if the alternative solution has the best condition.
Step 7: Rank the alternatives according to their value from S_{iw} $(i=1,2,\ldots,m)$.

Normalization

Two methods of normalization that have been used to deal with incongruous criteria dimensions are linear normalization and vector normalization.

Linear normalization can be calculated as in *Step 2* of the TOPSIS process above. Vector normalization was incorporated with the original development of the TOPSIS method (Hwang et al., 1987), and is calculated using the following formula:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum x_{ij}^2}}$$

for $i=1,2\ldots,m; j= 1,2,\ldots n$

In using vector normalization, the non-linear distances between single dimension scores and ratios should produce smoother trade-offs (Huang et al., 2011).

Technology

The procedures listed for the steps require technology in order to remove the tediousness of the calculations required. We have found examples produced in Excel as well as packages produced to perform TOPSIS such as SDI Tools.

Examples with TOPSIS

Example 1

We have four possible alternatives $\{A_1, A_2, A_3, A_4\}$ to choose from and each alternative has six attributes $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ and in this example the decision matrix contains only real values. Furthermore, all the entries in the decision maker imply larger values are better for decision purposes.

Step 1: The matrix **D.**

$$D = \begin{matrix} 2 & 1500 & 20000 & 5.5 & 5 & 9 \\ 2.5 & 2700 & 18000 & 6.5 & 3 & 5 \\ 1.8 & 2000 & 21000 & 4.5 & 7 & 7 \\ 2.2 & 1800 & 20000 & 5 & 5 & 5 \end{matrix}$$

Step 2: The normalized decision matrix using

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum x_{ij}^2}}$$

for $i=1,2\ldots,m; j= 1,2,\ldots n$ in Table 1.

Step 3: Weights

Step 3a: Assume that the weights provided by the decision maker are given as follows for $\{x_1\text{-}x_6\}$ as, 0.1, 0.15, 0.25, 0.1, 0.15, 0.25

Step 3b: If weights are not given by the decision maker then use Saaty's pairwise comparison for attributes and find the eigenvector of the consistent matrix. We will illustrate this next in this chapter.

Step 3c: Multiply the weights found in 3a or 3b by the matrix in Step 2 (see Table 2).

Step 4: In each column determine the maximum and minimum value for each column where larger is better. If any column's values represent smaller is the better result then we reverse the maximum and minimum procedures, respectively (Table 3).

Step 5: Compute S^+ and S^- for each alternative (see Table 4).

Step 6: Compute for each alternative the relative closeness to the ideal solution, $C= S^-/(S^+ + S^-)$ (Table 5).

Step 7. Rank the values in *Step 6* (Table 6).

A1 is best followed by A3, A2, and A4.

Example 2

Assume that the decision maker weights are not provided for the six attributes from the previous example. This may be where we decide to use Saaty's pairwise comparison. In Saaty's pair-wise comparison we must:

- Compare the relationship between two elements that share a common alternative in the hierarchy
- Comparisons ask 2 questions:
  - Which is more important with respect to the criterion or attribute?
  - How strongly?
- Build a matrix that shows results of all such comparisons
- Uses Saaty's 1-9 scale (Table 7)

Table 1. Normalized decision matrix

	x1	x2	x3	x4	x5	x6
A1	0.467142	0.366181	0.50556	0.506853	0.481125	0.67082
A2	0.583927	0.659125	0.455004	0.599008	0.288675	0.372678
A3	0.420428	0.488241	0.530838	0.414698	0.673575	0.521749
A4	0.513856	0.439417	0.50556	0.460776	0.481125	0.372678

Table 2. Modified matrix

	0.1	0.15	0.25	0.1	0.15	0.25
A1	0.467142	0.366181	0.50556	0.506853	0.481125	0.67082
A2	0.583927	0.659125	0.455004	0.599008	0.288675	0.372678
A3	0.420428	0.488241	0.530838	0.414698	0.673575	0.521749
A4	0.513856	0.439417	0.50556	0.460776	0.481125	0.372678

Table 3. Maximum and minimum values

Maximum	0.058393	0.098869	0.13271	0.059901	0.101036	0.167705
Minimum	0.042043	0.054927	0.113751	0.04147	0.043301	0.093169

Table 4. S+ and S- values for alternatives

	S+	*S-*
A1	0.055004	0.081581
A2	0.096168	0.050378
A3	0.051507	0.073599
A4	0.088063	0.034961

Table 5. Relative closeness to ideal solution

	C
A1	0.597289
A2	0.343767
A3	0.588296
A4	0.28418

Table 6. Ranked values

	C	Ranks
A1	0.597289	1
A2	0.343767	3
A3	0.588296	2
A4	0.28418	4

Table 7. Saaty's 9 Point scale

Intensity of Importance	Definition
1	Equal Importance
3	Moderate Importance
5	Strong Importance
7	Very Strong Importance
9	Extreme Importance
2,4,6,8	For compromises between the above

- Requires $n(n-1)/2$ judgments
- Inconsistency may arise that must be dealt with in the procedure.

Assume we obtain judgments by experts as to the relative pair-wise comparison importance of the six attributes (Table 8).

We check and find the matrix is consistent. Saaty defined the consistency index, CI as

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

If the referee is not absolutely consistent then in the above equation, $\lambda_{max} > n$, so we need to measure this level of inconsistency. For this purpose, Saaty defined a consistency ratio, *CR* as

CR=CI/RI

where *RI* is the average value of *CI* for random matrices using the Saaty scale obtained. We accepts a matrix as a consistent one if and only if CR < 0.10. To determine consistency we must compute or estimate the largest positive eigenvalue of our matrix and use it to find *CR*. In our example, we find the $CR = 0.0851$, which is less than 0.10. Thus, we continue to find the eigenvectors (decision weights) for our largest eigenvalue. The eigenvectors values are:

x1: 0.177970845
x2: 0.157764761
x3: 0.486816742
x4: 0.080912238
x5: 0.048875425
x6: 0.047659989

Table 8. Pairwise Comparisons where 1 equal 3 moderate 5 strong 7 very strong 9 extreme

	x_1	x_2	x_3	x_4	x_5	x_6
x_1	1/1	1/2	1/3	8	6	5
x_2	2	1/1	1/5	3	5	4
x_3	3	5	1/1	4	9	8
x_4	1/8	1/3	¼	1	1/2	2
x_5	1/6	1/5	1/9	2	1	1
x_6	1/5	1/4	1/8	1/2	1	1

Using these as the new decision weights then we obtain the new solution shown in Table 9.

With our new weights, *A1* is the best alternative followed by A4, A3, and A2.

Thus, of key importance are the decision maker's weights.

We point out another issue here. There is not a closed form for sensitivity analysis to measure the impact of differing weights on the rank ordering. One can use a trial and error process and we recommend that it be done in all analysis involving TOPSIS and the decision maker weights.

The evaluation matrix, **D**, can be an issue worth consideration. In Example 1 the values were real values that could have represented time, money, etc. Methods that we see in the literature for building the matrix appear to follow using real values or Saaty's 9-point matrix from the Analytical Hierarchy Process when there is either subjective attributes or attributes where bigger "real" values do relate to "better results." Thus, obtaining good values for the matrix D is important.

Example 3: Choosing a New Car Using TOPSIS

Let's consider choosing a new car from among a Honda Civic, Toyota Prius, Ford Focus, and Chevy Cruise. Further let's only examine the following attributes of these cars: style, reliability, fuel economy, and safety. These attributes, other than fuel economy, are subjectively measured. We examine each attribute separately. Style: Ford Focus, Toyota Prius, Honda Civic, and Chevy Cruise are in order of strength of style. We decide to give points are follows: 9-Ford Focus, 8-Toyota Prius, 7- Honda Civic, 6-Chevy Cruise. Next reliability where after examining consumer reports we give 9-Civic,7-Prius, 7-Cruise, 6-Focus. Fuel economy is to be measured in estimated mpg in the city: 33

Table 9. Solution after new decision weights

Weights	**0.177971**	**0.157765**	**0.486817**	**0.080912**	**0.048875**	**0.04766**
A1	*0.083138*	*0.05777*	*0.246115*	*0.041011*	*0.023515*	*0.031971*
A2	*0.103922*	*0.103987*	*0.221504*	*0.048467*	*0.014109*	*0.017762*
A3	*0.074824*	*0.077027*	*0.258421*	*0.033554*	*0.032921*	*0.024867*
A4	*0.091451*	*0.069324*	*0.246115*	*0.037282*	*0.023515*	*0.017762*
Maximum	0.103922	0.103987	0.258421	0.048467	0.032921	0.031971
Minimum	0.074824	0.05777	0.221504	0.033554	0.014109	0.017762
A1	0.053511		0.03195	A1	0.373855	4
A2	0.043803		0.056613	A2	0.563785	1
A3	0.04297		0.046239	A3	0.518322	2
A4	0.043862		0.033438	A4	0.432574	3

Table 10. Values for Matrix D

	x1	x2	x3	x4
A1	7	9	33	8
A2	8	7	40	7
A3	9	6	30	9
A4	6	7	35	6

Table 11. Saaty's method

	1	2	3	4
Style	**1**	1/4	1/9	1/5
reliability	4	**1**	1	4
fuel economy	9	1	**1**	3
Safety	5	1/4	1/3	**1**

mpg-Civic, 40 mpg,-Prius, 30 mpg-Focus, 35 mpg-Cruise. For safety, we again consult consumer's report. 9-Focus, 8-Civic, 7-Prius, 6-Cruise.

We have the values for our matrix D in Table 10.

We used Saaty's method to obtain our weights (Table 11, Table 12, Table 13).

Civic, Prius, Cruise, and Focus is the order of our car alternatives.

Weights, as we have seen in examples 1 and 2, are also an issue since the weights may either be chosen by the decision maker or found computationally through the decision maker weights methods via Saaty's 9 point scale from AHP. Weights, as we have shown, might significantly affect the ranking results. Again, the need for sensitivity analysis in criterion weights is deemed essential.

SOLUTIONS AND RECOMMENDATIONS

AHP has been a controversial technique in the operations research community. Harker and Vargas (1990) show that AHP does have an axiomatic foundation, the cardinal measurement of preferences is fully represented by the eigenvector method, and the principles of hierarchical compositionand rank reversal is valid. On the other hand, Dyer (1990a, 1990b) has questioned the theoretical basis underlying AHP and argues that it can lead to preference reversals based on the alternativeset being analyzed. In response, Saaty (1990) contends that rank reversal is a positive feature, when new reference points are introduced.

TOPSIS has been shown to be one of the best MADM methods in addressing the rank reversal issue, which is the change in the ranking of alternatives when a non-optimal alternative is introduced (Zanakis, Solomon, Wishart, & Dublish, 1998). This consistency feature is largely appreciated

Table 12. Eigenvector criterion weights

Style	0.065181761
Reliability	0.402574942
Fuel economy	0.404100918
Safety	0.128142379

Table 13. Final results and ranking

	C	Ranks
Civic	0.653318	1
Prius	0.546397	2
Ford	0.110291	4
Cruise	0.427366	3

in practical applications. Moreover, the rank reversal in TOPSIS is insensitive to the number of alternativesand has its worst performance only in the case of a very limited number of attributes (Triantaphyllou & Lin, 1996; Zanakis et al., 1998). A relative advantage of TOPSIS is its ability to identify the best alternative quickly (Paxkan & Wu, 1997).

FUTURE RESEARCH DIRECTIONS

Some future directions including comparative studies among AHP, DEA, and TOPSIS as well as other MADM, MCMD approaches that may be developed in the future. Knowing under what conditions these approaches are similar or yield the same in results would be extremely beneficial. Research into sensitivity analysis especially in criterion weights and there effect on final ranking would be useful.

The use of TOPSIS as a utility function to obtain outcomes for game theory strategies is an area of future study. MADM can be useful in ranking nodes in social network analysis (Fox, 2012 presentation).

Many papers in the literature suggest further research of using TOPSIS in the areas already mentioned but also in subject and topics yet to be determined.

CONCLUSION

We have presented and illustrated the technique of TOPSIS in MADM and MCDM analysis. Further we have pointed out some issues and controversies that could lead to future research. We have already seen in the literature hybrid analysis using AHP and TOPSIS together. We also find numerous applications of TOPSIS in business, industry, and government.

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KEY TERMS AND DEFINITIONS

Analytical Hierarchy Process: (AHP) is a technique created by Saaty using a 9 point scale to rank alternatives in a decision process and is useful to get decision maker weights for use in TOPSIS.

Decision Matrix: This is the *m x n* matrix of the *m* alternatives by *n* attributes.

Decision Weights (Eigenvectors): These are the subjective decision weights that are either provided by the decision maker or computed from the pair-wise comparison matrix as eigenvectors to the maximum eigenvalue.

Ideal Solution: Although assume unachievable the ideal and negative ideal solution are used to compute the ratios of distances from the ideal and negative ideal solution.

Normalization Process: The normalization process for TOPSIS differs from others process in that TOPSIS considered distances.

S^-: This represents the distance of the computed value to the negative ideal solution.

S^+: This represents the distance of the computed value to the ideal solution.

TOPSIS: Technique for order preference by similarity to ideal solution.